



From the Red Room

Artillery Expression and Cleaning Up the Math

David B. Nance & Captain Seth E. Robinson

Rounding, banker's rounding, truncation, and Artillery Expression are all mathematical methods used to manage decimal fractions in interpolation and firing solutions. While each has merit, the appropriate method depends on context and required precision. One of the most frequently asked questions received by the Gunnery Department concerns the correct application of Artillery Expression; a method widely recognized within the Artillery community but inconsistently applied due to doctrinal ambiguity and deviation from original intent.

Current doctrinal references, such as *TC 3-09.81* and other legacy materials, fall short in providing the clear, actionable guidance necessary for uniform application. Although examples of Artillery Expression appear sporadically, they lack consistency and do not address modern precision demands or the nuances of expression for values beyond neat 0.5 increments. Without doctrinal clarity, inconsistency across units erodes effectiveness, accuracy, and shared understanding.

The Problem with Traditional Rounding

Traditional rounding is commonly taught in early education and operates on a straightforward rule: values of 0.4 and below round down, while 0.5 and above round up. For example:

- $2.5 \rightarrow 3$
- $3.4 \rightarrow 3$

This simplicity becomes a liability in artillery applications as in other precision-oriented disciplines such as engineering and finance. In manual or degraded environments, where every mil matters, rounding errors can introduce significant inaccuracies in firing solutions. Individual errors can range from nearly 0 to 0.5, with an upward bias due to the rounding of values ending in 0.5. This may result in overshooting a target, which is particularly problematic due to the forward and lateral fragmentation pattern of artillery rounds (*ATP 3-09.80*, Appendix B-3, Figure B-2).

Spc. Damien Galvan, a cannon crewmember assigned to Charlie Battery, 2nd Battalion, 77th Field Artillery Regiment, 4th Infantry Division Artillery, 4th Infantry Division, confirms coordinates through a M777 howitzer gun scope during Ivy Sting 5 on Fort Carson, Colorado, March 10, 2026. Ivy Sting 5 was a multi-domain training exercise where units practiced large scale combat operations while integrating modern command and control capabilities. (U.S. Army photo by Pfc. Jacob Cruz)

Artillery Expression: A Superior Alternative

Artillery Expression mitigates rounding inaccuracies by employing banker’s rounding: rounding 0.5 values to the nearest even number. This technique is widely used in financial systems due to its ability to reduce cumulative rounding bias over repeated calculations.

Example Using Artillery Expression:

Traditional Rounding	Raw Data	Artillery Expression
+1	+0.5	0.0
+2	+1.5	+2.0
+3	+2.5	+2.0
+4	+3.5	+4.0
+5	+4.5	+4.0
Sum: 15	Sum: 12.5	Sum: 12

The accuracy gain over traditional rounding is both measurable and operationally relevant. Although individual errors may still range from nearly 0 to 0.5, Artillery Expression can reduce the total expected error across a dataset by mitigating the upward bias inherent in traditional rounding. This method is not unfamiliar. It has even appeared in pop culture, notably in the plots of *Superman III* (1983) and *Office Space* (1999), as a means of rounding financial fractions for gain. Its practical value lies in eliminating long-term error trends.

Real-World Values and Doctrinal Gaps

Most real-world values do not terminate neatly in 0.5. For instance, values like 2.59, 3.56, or 2.50000000001 raise questions of precision and significance. Current doctrine does not provide adequate guidance for how these should be expressed. TC 3-09.81 (*April 2016*) mentions Artillery Expression only a scant three times, providing just one concrete use example (PG 7-27: L10.5 → L10). Tabular Firing Table AR-2 (*2016*) dedicates only three sentences to the concept. These documents fail to address expressions involving multiple significant figures or complex decimals. The tabular firing tables do offer insight:

- **Nearest whole number:** round to the nearest even number when the value ends in 0.5
 - **Nearest tenth:** round to the nearest even number when the value ends in 0.05
 - The same logic applies to hundredths, thousandths, and so on
- While beneficial, comprehensive doctrinal guidance remains absent.

The Truncation vs Artillery Expression Debate

The inconsistency worsens when considering how computational devices treat decimals. For example:

- iPhones and TI calculators round the final digit (e.g., $2 \div 3 \rightarrow 0.6666667$)
- Google Calculator truncates or drops the final digit (e.g., 0.666666)

These divergent approaches could create two camps within the Artillery community: those who Artillery express and those who simply truncate.

Comparison of Averages Example:

Truncation	Raw Data	Artillery Expression
+0	+0.51	+1
+5	+5.76	+6
+2	+2.09	+2
+4	+4.58	+5
+11	+12.94	+14
Cumulative error using truncation: 1.94		
Cumulative error using Artillery Expression: 1.06		

Artillery Expression more often produces more accurate data than truncation and seems the superior alternative. Statistically, truncation is an unbiased method as it does not favor expressing in either direction, however when applied to a large dataset it introduces a downward systematic bias by always resulting in a number lower than the original value. In artillery fire direction, where precision is critical, truncation ensures that calculations remain consistent and predictable, particularly in isolated decision points where each firing solution is treated independently. By eliminating the rounding step, truncation simplifies computations and reduces the risk of introducing additional error, making it a reliable method for expressing firing data in time-sensitive environments. But is there a method that can maintain the accuracy produced by Artillery Expression and harness the ease of computing of truncation?

Significant Figures Artillery Expression

In an operational context, the ease-of-use advantage of truncation remains significant, but the accuracy of Artillery Expression remains. What matters instead is how quickly and consistently Soldiers can apply the method under pressure. Artillery Expression offers a clear advantage here. It mirrors the intuitive logic of traditional rounding, avoids the systematic downward drift of truncation, and can be applied more rapidly without sacrificing accuracy. Artillery Expression provides the right balance of precision, speed, and doctrinal clarity but can we enhance it as currently practiced in a better way? When combined with the discipline of significant figures, ensuring that results are expressed only to the level of accuracy the system supports, Artillery Expression becomes not just a rounding method, but a complete standard for precision in firing data. Utilizing significant figures harnesses the ease of truncation but ensures that the level of accuracy remains efficient for the system that is being utilized.

Other precision-dependent disciplines have long recognized the need to combine significant figures with a consistent rounding method, and they enforce this through standards and training. In finance, for example, accounting systems

$$100 \left\{ 70 \left\{ \begin{array}{cc} 5600 & 449.3 \\ 5670 & ??? \\ 5700 & 462.4 \end{array} \right\}^x \right\} 13.1$$

$$70/100 = x/13.1$$

$$X = (70)(13.1)/100$$

$$X = 9.17 \approx 9.2$$

$$449.3 + 9.2 = 458.5 \approx 458$$

Figure 3. Current Artillery Expression Example

apply banker's rounding at the last digit to prevent cumulative bias, while significant figures ensure that currency is never expressed beyond the cent. In engineering and manufacturing, measurements are carried at full precision internally, but final drawings and tolerances are expressed only to the number of digits the instruments can reliably measure.

Similarly, aviation and navigation systems compute headings and positions to high precision but present them to pilots at a doctrinally standardized level, such as the nearest degree or tenth of a mile. Across these fields, the pattern is consistent: maintain maximum precision during computation, then apply significant figures and then banker's rounding together at the final step. This ensures that results are both mathematically accurate and operationally useful, a model that Artillery can adopt to improve consistency and lethality.

In light of these lessons, Artillery must move away from the outdated habit of "expressing at the X", rounding or truncating results at every intermediate step of a calculation. This step is nondoctrinal and is an erroneous change (informally named a Square []) that is added within the institution that has no basis on foundational and proven mathematical principles. This practice introduces unnecessary errors and false precision, while offering no operational benefit. Other disciplines have already solved this problem: finance, engineering, and aviation all carry computations at full precision internally and only apply significant figures and rounding once, at the final output. By adopting the same standard, Artillery ensures that firing data remains mathematically sound throughout the calculation process, while still producing a final expression that is both precise and doctrinally uniform.

Figures 3 and 4 illustrate the critical difference between the current practice of "expressing at the X" and the proposed method of maintaining full computational precision until the final step. In the first figure (current method), intermediate results are expressed or at each step of the calculation, introducing cumulative errors that compound with each operation. This leads to a final result that deviates from the

true value, even though the errors at each step may seem small. While in this example the end result remains the same, the additional erroneous expression wastes time and increases cognitive load for the user.

In contrast, the second figure demonstrates the proposed method, where all intermediate calculations are carried out at full precision. Expression is applied only once, at the final step, using significant figures and Artillery Expression. This approach eliminates cumulative errors, ensuring that the final result is both mathematically accurate and operationally precise. By comparing the two methods, the figures highlight how the proposed approach minimizes error, avoids false precision, and aligns with doctrinal principles, ultimately improving the accuracy and consistency of firing solutions and reducing user cognitive loads while under duress.

Implications for Doctrine and Training

In degraded environments, manual computation is essential. Any unnecessary error, especially one introduced by an outdated or misunderstood rounding technique, reduces unit lethality and introduces risk. This is not just a mathematical issue; it is a doctrinal failure. Training institutions, from AIT to BOLC to NCO academies, must adopt a single, standardized method of Artillery Expression. The inconsistent use of truncation and bad Artillery Expression (express at the X) across the force hampers effectiveness and creates unit to unit friction during joint operations, training exercises, and fire missions. It is imperative that *TC 3-09.81* and related documents be updated to clearly define Artillery Expression as a distinct and uniform method, separate from truncation, and applied consistently across all platforms.

Conclusion and Recommendation

To ensure a lethal, competent, and mathematically precise artillery force, we must return to the original intent of Artillery Expression. By removing truncation, eliminating expression at

$$100 \left\{ 70 \left\{ \begin{array}{cc} 5600 & 449.3 \\ 5670 & ??? \\ 5700 & 462.4 \end{array} \right\}^x \right\} 13.1$$

$$70/100 = x/13.1$$

$$X = (70)(13.1)/100$$

$$X = 9.17$$

$$449.3 + 9.17 = 458.47 \sim 458$$

Figure 4. Significant Figure Artillery Expression Example |

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the X, and standardizing even-digit rounding through Artillery Expression principles along with the foundational math principle of significant figures, we realign the standards taught in FABOLC, AIT and the NCO academy. This alignment reduces friction between Enlisted and Officers emerging from the schoolhouse, while ensuring greater consistency, reduced error, and improved lethality.

Recommendations:

- Revise TC 3-09.81 to define Artillery Expression explicitly.
- Eliminate truncation from doctrinal examples and instructional materials across all generating and operational forces.
- Eliminate the unnecessary and nondoctrinal express at the X.
- Integrate Artillery Expression instruction into all Field Artillery schools and certification programs.

A shared understanding of Artillery Expression is critical to mission success. It is time to update doctrine, align training, and reinforce precision across the force.

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